Name____ Date _______________

Writing Equations to Model Data

When data have equally-spaced inputs, you can analyze patterns in the differences of the outputs to determine what type of function can be used to model the data. Linear data have constant *first differences*. Quadratic data have constant *second differences*. The first and second differences of $f(x) = x^2$ are shown below.

Equally-spaced x-values

x	−3	−2	−1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

first differences: −5 −3 −1 1 3 5

second differences: 2 2 2 2 2

Time, t	Height, h
10	26,900
15	29,025
20	30,600
25	31,625
30	32,100
35	32,025
40	31,400

NASA can create a weightless environment by flying a plane in parabolic paths. The table shows heights h (in feet) of a plane t seconds after starting the flight path. After about 20.8 seconds, passengers begin to experience a weightless environment. Write and evaluate a function to approximate the height at which this occurs.

SOLUTION

Step 1 The input values are equally spaced. So, analyze the differences in the outputs to determine what type of function you can use to model the data.

$h(10)$	$h(15)$	$h(20)$	$h(25)$	$h(30)$	$h(35)$	$h(40)$
26,900	29,025	30,600	31,625	32,100	32,025	31,400

First differences: 2125 1575 1025 475 −75 −625

Second differences: −550 −550 −550 −550 −550

Because the second differences are constant, you can model the data with a quadratic function.

Step 2 Write a quadratic function of the form $h(t) = at^2 + bt + c$ that models the data. Use any three points (t, h) from the table to write a system of equations.

Use (10, 26,900): $100a + 10b + c = 26{,}900$ Equation 1

Use (20, 30,600): $400a + 20b + c = 30{,}600$ Equation 2

Use (30, 32,100): $900a + 30b + c = 32{,}100$ Equation 3

Use the elimination method to solve the system.

Step 3 Evaluate the function when $t = 20.8$.

Real-life data that show a quadratic relationship usually do not have constant second differences because the data are not *exactly* quadratic. Relationships that are *approximately* quadratic have second differences that are relatively "close" in value. Many technology tools have a *quadratic regression* feature that you can use to find a quadratic function that best models a set of data.

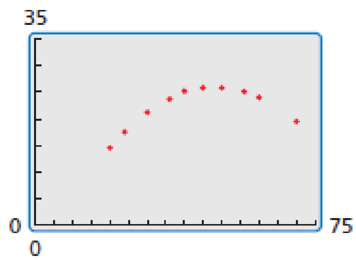
The table shows fuel efficiencies of a vehicle at different speeds. Write a function that models the data. Use the model to approximate the optimal driving speed.

Miles per hour, x	Miles per gallon, y
20	14.5
24	17.5
30	21.2
36	23.7
40	25.2
45	25.8
50	25.8
56	25.1
60	24.0
70	19.5

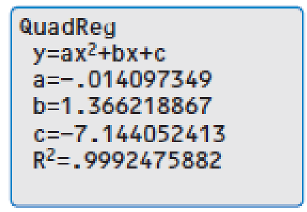
SOLUTION

Because the x-values are not equally spaced, you cannot analyze the differences in the outputs. Use a graphing calculator to find a function that models the data.

Step 1 Enter the data in a graphing calculator using two lists and create a scatter plot. The data show a quadratic relationship.

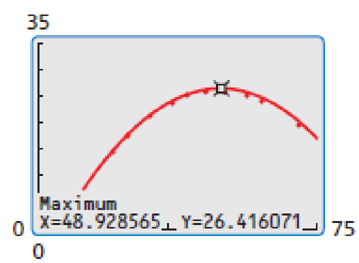


Step 2 Use the *quadratic regression* feature. A quadratic model that represents the data is $y = -0.014x^2 + 1.37x - 7.1$.



Step 3 Graph the regression equation with the scatter plot.

In this context, the "optimal" driving speed is the speed at which the mileage per gallon is maximized. Using the *maximum* feature, you can see that the maximum mileage per gallon is about 26.4 miles per gallon when driving about 48.9 miles per hour.



▶ So, the optimal driving speed is about 49 miles per hour.

Write an equation of the parabola that passes through the points $(-1, 4)$, $(0, 1)$, and $(2, 7)$.

The table shows the estimated profits y (in dollars) for a concert when the charge is x dollars per ticket. Write and evaluate a function to determine what the charge per ticket should be to maximize the profit.

Ticket price, x	2	5	8	11	14	17
Profit, y	2600	6500	8600	8900	7400	4100

The table shows the results of an experiment testing the maximum weights y (in tons) supported by ice x inches thick. Write a function that models the data. How much weight can be supported by ice that is 22 inches thick?

Ice thickness, x	12	14	15	18	20	24	27
Maximum weight, y	3.4	7.6	10.0	18.3	25.0	40.6	54.3